NewFrac
Training Network

# WORKSHOP 1
# NEW STRATEGIES IN COMPUTATIONAL FRACTURE MECHANICS

New strategies for multifield fracture problems across scales in heterogeneous systems for energy, health and transport

School of Engineering of the University of Seville
October 1st – 8th, 2021

MARIE CURIE ACTIONS

# Dialogue between Finite Fracture Mechanics and Phase field approach for brittle fracture

A. Doitrand[1], G. Molnar[2], R. Estevez[3], A. Gravouil[2]

[1] Univ Lyon, INSA Lyon, UCBL, CNRS, MATEIS, UMR5510, 69621 Villeurbanne, France
[2] Univ Lyon, INSA Lyon, UCBL, CNRS, LAMCOS, UMR5259, 69621 Villeurbanne, France
[3] Univ Grenoble Alpes, Grenoble INP, CNRS, UMR5266, SIMaP, 38000, Grenoble, France

aurelien.doitrand@insa-lyon.fr

**Abstract**: Phase-field approach for fracture and the coupled criterion (CC) attracted much attention in recent years due to their ability to model fracture. Phase-field model consists in diffusing the crack surface into the volume of the solid, thus making its implementation possible through variational techniques. This diffusion is controlled by an internal length parameter, which was primarily considered to be a numerical aid without any real physical meaning. In addition to this internal length, the material critical energy release rate is required as input of the phase-field model. The latter is also an input parameter of the CC as well as the material tensile strength.

The CC was originally developed in order to overcome the limitation of Linear Elastic Fracture Mechanics to assess crack initiation [1]. Phase-field approach also enables modeling the initiation of a crack. Therefore, the same fracture problem can be modeled with both approaches. We thus question the consideration that the internal length is only a numerical parameter and try to establish a correlation between the internal length of the phase field model and the material tensile strength.

Both approaches are confronted based on several benchmark examples [2]. The correlation between both methods and thus between their input parameters is demonstrated based on the critical loading, the crack topology and the crack arrest length. Independently of the chosen aspect, the phase-field approach and the CC present excellent correspondence. The correlation between the material tensile strength and internal length appears unique. Interestingly, we find that both stress and energy criteria are satisfied in the phase-field fracture, which is explained by demonstrating the alteration in global energy release rate due to the regularization introduced by the smeared model.

**References:**

[1] Leguillon D. 2002. Strength or toughness? A criterion for crack onset at a notch. European Journal of Mechanics - A/Solids, volume 21 (1): 61–72.

[2] Molnar G, Doitrand A, Estevez R, Gravouil A. 2020. Toughness or strength? Regularization in phase-field fracture explained by the coupled criterion. Theoretical and Applied Fracture Mechanics, volume 109: 102736.

# Solidification of a water drop – A paradox

Oriana Haddad,  Dominique Leguillon

Institut Jean Le Rond d'Alembert
CNRS UMR 7190 – Sorbonne Université
4 place Jussieu 75005 PARIS – France

dominique.leguillon@upmc.fr

**Abstract**: When a liquid water drop falls on a very cold plate, ice is formed almost instantly after the impact on the cold surface. If the plate is cold enough, cracks may appear in the ice layer. Depending of the temperature $T_c$ of the plate, three cracking regimes were observed in experiments [1]. For $T_c >$ -20 °C, a first regime shows that the water solidifies into ice without creating cracks. A second regime appears around $T_c$ = -30 °C, the water solidifies entirely into ice, then suddenly closely spaced cracks are formed. A third regime is presented for $T_c <$ -30 °C, cracks appear following a hierarchical order while the water is still solidifying, these cracks are significantly more spaced than in the second regime.

These three mechanisms are paradoxical. Indeed, the plate exerts a thermal loading, the colder the plate, the more intense the loading. We would therefore expect multi-cracking to be more dense for higher loadings, and the opposite is observed. Based on a simplified 2D model, the Coupled Criterion [2,3] is able to explain this surprising phenomenon.

**References:**

[1] E. Ghabache, C. Josserand, T. Séon, Frozen Impacted Drop: From Fragmentation to Hierarchical Crack Patterns, Physical Review Letters 117, 074501 (2016).

[2] D. Leguillon, Strength or toughness? A criterion for crack onset at a notch, Eur. J. of Mechanics – A/Solids 21, 61-72 (2002).

[3] D. Leguillon D., E. Martin. Prediction of multi-cracking in sub-micron films using the coupled criterion. Int. J. Fract., 209, 187-202 (2018).

# Finite element implementation of CCFFM based on PMTE-SC to predict crack onset and growth in composites

A.S. Karthik[1], V. Mantič[1], M. Paggi[2], M. Muñoz-Reja[1], L. Távara[1]

[1]Grupo de Elasticidad y Resistencia de Materiales, Escuela Técnica Superior de Ingeniería, Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain
[2]IMT School for Advanced Studies Lucca, Piazza San Francesco 19, 55100, Lucca, Italy
karthik@us.es

**Abstract**: The numerical prediction of fracture in composites is a classical challenge in both solid mechanics and computational science. In the past two decades, extensive research has been conducted to develop effective and accurate numerical models for simulating fracture behaviour. The main objective of this work is to present the numerical efforts for developing algorithms to predict crack onset and growth in composites. This research focuses on the simulation of the two-dimensional cracking process in brittle materials based on the Coupled Criterion of Finite Fracture Mechanics (CCFFM). In the framework of the Finite Fracture Mechanics (FFM), assuming crack advances by finite steps, in opposite to the hypothesis of crack advance by infinitesimal steps adopted in the classical Linear Elastic Fracture Mechanics (LEFM), Leguillon proposed the coupled criterion which requires both stress and energy conditions are simultaneously fulfilled [1].

In the present work, the prospective crack path is modelled by a discontinuous representation of crack bridged by a continuous distribution of stiff springs with a linear elastic behaviour up to its breakage, which is a similar approach as used previously in Linear Elastic-(perfectly) Brittle Interface Model (LEBIM) [2]. The methodology of the current work is the application of CCFFM to the spring model based on the Principle of Minimum Total Energy subjected to a Stress Condition (PMTE-SC), where the energy criterion is imposed by minimizing the total energy change due to a crack advance. This new formulation of the CCFFM was introduced by Mantič [3], who analyzed several aspects of the application of the PMTE-SC to the crack onset and propagation. The CCFFM based on the PMTE-SC is more versatile than the approach based on the stress and energy criteria curves, which is usually used at present. For this reason, the implementation of the CCFFM by PMTE-SC in FEM could provide a tool capable of solving complex fracture problems [2].

The FEM has been widely used to solve different problems in the field of fracture mechanics. Apart from the standard FEM along with remeshing and adaptive meshing techniques, new methods have been developed to improve the accuracy of the solution in 2D linear elastic fracture mechanics problems, in the last two decades, such as the extended FEM (XFEM) or the phantom node method (PNM). However, these techniques require additional DOFs as well as enriched elements which results into problems with transition elements in the case of XFEM, whereas in the case of PNM, lack of crack tip functions causes non-asymptotic stress field in crack tip [4]. In addition, the implementation of CCFFM based on PMTE-SC in these recent techniques is a computationally intensive procedure. Therefore, the simplest implementation is to simulate crack propagation along the element edges, in which the cracks are geometrically modelled as topological discontinuities, i.e., cracks are introduced explicitly during the discretization of the domain, matching the faces of the elements with the crack faces. In this work, we introduce the implementation within the FEM software ABAQUS using some of the subroutines such as UMAT, USDFLD, etc.

**References:**

[1] D. Leguillon. 2002. Strength or toughness? a criterion for crack onset at a notch. European Journal of Mechanics A/Solids, 21, 61–72.

[2] M. Muñoz-Reja, L. Távara, V. Mantič, P. Cornetti.2016. Crack onset and propagation at fibre-matrix elastic interfaces under biaxial loading using finite fracture mechanics. Composites Part A, 82, 267–278.

[3] V. Mantič. Prediction of initiation and growth of cracks in composites. 2014. Coupled stress and energy criterion of the finite fracture mechanics. 16th European Conference on Composite Materials (ECCM 2014).

[4] M. Marco, D. Infante-García, R. Belda, E. Giner. 2020. A comparison between some fracture modelling approaches in 2D LEFM using finite elements. International Journal of Fracture, 223, 151-171.

# Study of instabilities due to crack propagation

Simone Sangaletti[1], Israel Garcia Garcia [1]

[1] Grupo de Elasticidad y Resistencia de Materiales, Escuela Técnica Superior de Ingeniería,Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain

ssangaletti@us.es

**Keywords:** *Phase field, Control algorithm, Snap-back, Fracture Mechanics*

**Abstract**: The computation of dissipated energy is a fundamental topic in the field of Fracture Mechanics since it is considered as a parameter fundamental in the design of structures. Said so, its computation is not trivial since, depending on the specimen or component geometry, instabilities due to crack propagation may arise, jeopardizing both the convergence of the simulation and the numerical computation of the dissipated energy, which is inevitably wrong if we just perform a traditional Finite Element analysis. Instabilities can be caught using some Force-Displacement control approaches which are already built in FE software like Arch-Length procedures. These are usually applied only for geometrical instabilities due to snap-backs not arising from crack propagation. It is then interesting to find the way to catch these phenomena when a crack propagation is involved, especially in unstable scenarios. In this work a Phase Field approach is applied to model the crack propagation in the specimen exploiting Abaqus finite element software and already available subroutines [1]. The control algorithm developed in [2] is coupled with Phase Field simulations in order to catch the snap-back consequent to crack propagation, trying to understand in which cases instabilities phenomena could arise and the possibilities to compute the dissipated energy numerically basing on the Force Displacement curve obtained from the simulation.

**References:**

[1] Y. Navidtehrani, C. Betegón, E. Martínez-Pañeda(2021)A Unified Abaqus Implementation of the Phase Field Fracture Method Using Only a User Material Subroutine.Materials 14 (8) 1913.

[2] E. Martínez-Pañeda, Susana del Bustob, Covadonga Betegón (2017) Non-local plasticity effects on notch fracture mechanics

# Finite Fracture Mechanics from the macro- to the micro-scale

Sara Jiménez Alfaro[1], José Reinoso [2], Dominique Leguillon[1]

[1] Institut Jean Le Rond d'Alembert. Sorbonne Université. CNRS UMR 7190. 4 Place Jussieu, 75000, Paris, France
[2] Escuela Técnica Superior de Ingenieria. Universidad de Sevilla. Avda. Descubrimientos, 41092, Seville, Spain

sara.jimenez_alfaro@sorbonne-universite.fr

**Keywords:** Finite Fracture mechanics – Coupled Criterion - Micro-scale – Fracture properties

**Abstract**: Within the framework of Finite Fracture Mechanics (FFM), one of the fundamental ingredients of this methodology is that a crack is assumed to jump a given finite length at onset. This can be formulated through the invocation of the Coupled Criterion (CC) [1], stating that this length depends on the material toughness, the tensile strength but also the geometry. Complying with a different vision, according to the Phase Field (PF) model [2] there exists a length related to the size of the damaged region, called the phase field length scale. Both the PF length scale and the nucleation length obtained by the CC are proportional to the Irwin length defined from the material toughness and tensile strength. At the macro-scale, they are small compared to any dimension of the structure, whereas at the micro-scale both lengths are of the same order of magnitude or even larger and can interact with the dimensions of the structure.

The aim of this work is to analyze how the answer brought by the CC and the PF model evolves when descending the scales from the cm-scale to the µm-scale and even nm-scale. Based on previous results, it can be argued that both the CC and the PF model provide satisfactory predictions of cracking events in solids. However, this can be a controversial issue at smaller scales of analysis due to a lack of energy because of the smallness of the specimens. This is attributed to the fact that at such scales it is seen that the corresponding results are much sensitive to the toughness but less sensitive to the tensile strength.
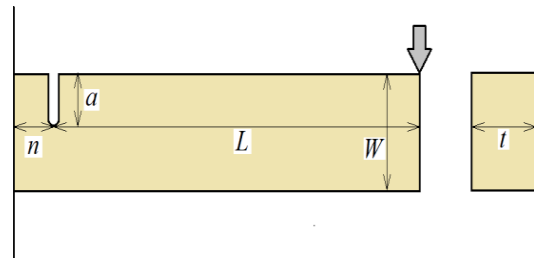


Figure 1: The U-notched micro-cantilever beam.

Relying on the previous discussion, in this contribution, bending tests on notched micro-cantilever beams (Figure 1) made of a ceramic material 8Y-FSZ cubic zirconia are investigated. Particularly, we analyze how the fracture nucleation is affected considering different values of the toughness and the strength, according to the CC. A wider scattering in the determination of the tensile strength than in that of the fracture energy is observed. Furthermore, a comparison of the critical load calculated using the CC, to the ones obtained in experiments [3] is made. Then, an optimization process is followed to estimate an average toughness, based on the minimization of the difference between numerical simulations and experiments. Finally, we replicate this analysis using the PF approach of fracture as modelling tool. In the latter, the influence of the phase field length scale in the crack nucleation at the micro-scale is examined, this parameter being considered a property of the material because of its relation to the strength [2].

## References:

[1] Leguillon, D. (2002). Strength or toughness? A criterion for crack onset at a notch. *European Journal of Mechanics-A/Solids*, *21*(1), 61-72.

[2] Tanné, E., Li, T., Bourdin, B., Marigo, J. J., & Maurini, C. (2018). Crack nucleation in variational phase-field models of brittle fracture. *Journal of the Mechanics and Physics of Solids*, *110*, 80-99.

[3] Henry, R., Zacharie-Aubrun, I., Blay, T., Chalal, S., Gatt, J. M., Langlois, C., & Meille, S. Fracture properties of an irradiated PWR UO2 fuel evaluated by micro-cantilever bending tests. J. Nuclear Materials 2020; 538: 152209.

# Numerical assessment of the effect
# of in-plane shear plasticity in the tensile response of a DEN composite plate

Anatoli A. Mitrou [1], Albertino Arteiro [1], Pedro P. Camanho [1]

[1] DEMec, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal

anatomitrou@fe.up.pt

**Abstract**: For the aerospace industry, the use of high-grade thermoplastics as a matrix for carbon fiber reinforced polymers (CFRPs) in key structural elements, instead of the conventionally used thermosets, is very appealing due to the increased toughness, better heat resistance, faster processing times and thus lower cost they offer [1][2]. Control surfaces made of high-grade thermoplastic CFRPs are already used by Gulfstream Aero, for example. Thermoplastics, though, are expected to have a more ductile response and it has been shown that damage formation and the mechanisms that take place differ significantly between these two material systems. Split cracks and delaminations dominate in thermosets, while shear failure dominates in thermoplastics [3]. Motivated by this interest in transitioning to the use of thermoplastics, this work aimed to demonstrate the effect of the level of in-plane shear plasticity of the matrix material in the response of a Double Edge Notched (DEN) composite plate.

Using the model from Furtado et al. [4], a purely numerical study was conducted to provide an answer to this question. In-plane shear plasticity parameters are altered in the constitutive relation for in-plane shear of the material system and the results for various degrees of ductility/plasticity are then compared. It was shown that increased ductility could potentially lead to overall strength reduction, something not necessarily expected and obvious. This was attributed to the difference in the observed damage mechanisms leading to failure. Lower levels of in-plane shear plasticity allowed for damage to grow and propagate sooner, allowing load redistribution at the notch tip earlier in the loading history. Higher levels of in-plane shear plasticity, on the other hand, delays matrix cracking and, consequently, load redistribution, effectively reducing the notched strength of the plate, as stress concentration is kept localized.

This initial parametric study certainly points out the need to further enhance the knowledge on the mechanical response and fracture behavior of thermoplastic matrix CFRPs. In that direction, further studies would be beneficial to address the effect of other parameters (e.g., the yield stress) as well as see whether the ply thickness affects the results in a similar way as was observed by Furtado et al. [4].

**References:**

[1] J. D. Muzzy, and A. O. Kays. 1984. Thermoplastic vs. thermosetting structural composites. Polymer Composites 5, no. 3, 169-172.

[2] D. Meyer, P. Carnevale, H. Bersee, and A. Beukers. 2009. New affordable reinforced thermoplastic composite for structural aircraft applications. In 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 17th AIAA/ASME/AHS Adaptive Structures Conference 11th AIAA No, p. 2337.

[3] K. Srinivasan, W. C. Jackson, B. T. Smith, and J. A. Hinkley. 1992. Characterization of damage modes in impacted thermoset and thermoplastic composites. Journal of reinforced plastics and composites 11, no. 10, 1111-1126.

[4] C. Furtado, A. Arteiro, P. Linde, B. L. Wardle, and P. P. Camanho. 2020. Is there a ply thickness effect on the mode I intralaminar fracture toughness of composite laminates?. Theoretical and Applied Fracture Mechanics, 107, 102473

# Fracture propagation as a standard dissipative process: application to hydraulic fractures

Francesca Fantoni [1], Alberto Salvadori [2]

[1] DICATAM, University of Brescia, via Branze 43, 25123 Brescia, Italy
[2] DIMI, University of Brescia, via Branze 38, 25123 Brescia, Italy
Alberto.salvadori@unibs.it

**Abstract**: Recent publications framed the problem of three-dimensional quasi-static crack propagation in brittle materials into the theory of standard dissipative processes [1]. Variational formulations, stated therein, characterize the three-dimensional crack front quasi-static velocity as the minimizer of constrained quadratic functionals. An implicit in time crack tracking algorithm, that computationally handles the constraint via the penalty method algorithm, was developed and implemented in [2].

Although the theoretical setting is sound, the derived crack tracking methods suffered from a major drawback that limited the interest in the method to its theoretical content. Specifically, the need of still currently unavailable accurate approximations for weight functions made the approach of minor interest from the numerical standpoint. Such a drawback was overcome in [3], where a viscous regularization of the fracture propagation in brittle materials as a standard dissipative process was formulated. Rate-dependency provided a simple and accurate approximation of the crack front velocity, thus allowing to formulate effective crack tracking algorithms.

That idea is further developed here to model hydraulic fracture processes [4]. Although limited to a penny shaped crack benchmark, the novel set of differential equations that are here proposed are capable to model the evolution of the lag and of the crack advancing in a straightforward way. The formulation can be easily extended to account for fractures pressurized by gas or other substances.

**References:**

[1] A. Salvadori. A plasticity framework for (linear elastic) fracture mechanics. J MECH PHYS SOLIDS, 56:2092–2116, 2008.

[2] A. Salvadori and F. Fantoni. Fracture propagation in brittle materials as a standard dissipative process: general theorems and crack tracking algorithms. J MECH PHYS SOLIDS, 95:681–696, 2016.

[3] A. Salvadori, P.A. Wawrzynek, and F. Fantoni. Fracture propagation in brittle materials as a standard dissipative process: Effective crack tracking algorithms based on a viscous regularization. J MECH PHYS SOLIDS, 127:221–238, 2019.

[4] E. Detournay. Mechanics of hydraulic fractures. ANNU REV FLUID MECH, 48:311–339, 2016

# Nucleation and propagation of cracks under multi-axial loading in phase-field modelling

Camilla Zolesi [1], Corrado Maurini [1], Laura De Lorenzis [2]

[1] CNRS, Institut Jean Le Rond d'Alembert, Sorbonne University, UMR 7190, 75005, Paris, France
[2] Department of Mechanical and Process Engineering, ETH Zürich, Tannenstrasse 3, 8092 Zürich, Switzerland

camilla.zolesi@sorbonne-universite.fr

**Keywords:** *damage, fracture, phase field, variational methods, finite elements*.

**Abstract**: The phase-field approach to brittle fracture is based on the following minimization problem

$$\min_{\boldsymbol{u},\alpha} \mathcal{E}_{tot}\,(\boldsymbol{u},\alpha) = \int_{\Omega} [\varphi(\text{s}(\boldsymbol{u}),\alpha\,) + \psi_{\text{diss}}(\alpha,\boldsymbol{\nabla}\alpha)]\mathrm{d}\Omega \qquad \text{with}\ \ \varphi(\text{s}(\boldsymbol{u}),\alpha) = \text{a}(\alpha)\varphi_R(\text{s}(\boldsymbol{u}),\alpha) + \varphi_D(\text{s}(\boldsymbol{u}),\alpha)$$

where $\mathcal{E}_{tot}$ is the total energy functional, $\boldsymbol{u}$, $\alpha$, $\text{s}(\boldsymbol{u})$ are respectively the displacement and damage fields and the strain tensor, and $\varphi$ and $\psi_{\text{diss}}$ are respectively the elastic and the dissipated energy density. The minimization problem is solved under the irreversibility constraint for the damage field. The elastic energy density is split into a damageable part ($\varphi_D$) and a residual part ($\varphi$), which serves the double purpose of avoiding interpenetration of the crack surfaces under compression, and reflecting the physical asymmetry of fracture behaviour between tension and compression.

Recent results have proved that phase-field approaches can quantitatively predict crack nucleation for mode-I loading [1]. However, the prediction of crack nucleation under multiaxial loading is crucially influenced by the energy decomposition, and currently available decompositions (e.g. those in [2,3]) are insufficiently flexible [4] as they do not allow to reproduce the experimentally measured tensile and compressive (or shear) strengths. To solve the issue, a novel decomposition was proposed in [4], giving a parametric strength surface à la Drucker-Prager to be adjusted based on the experimentally measured tensile and compressive strength.

In this contribution, the new theoretical model is implemented numerically to verify the analytical nucleation curves. Moreover, the analysis is extended beyond nucleation to analyse the localisation modes under multiaxial loading and to study the propagation behaviour. This implies dealing with the non-linearities introduced by the new model and with bad conditioning and locking issues related to the corresponding linear system.
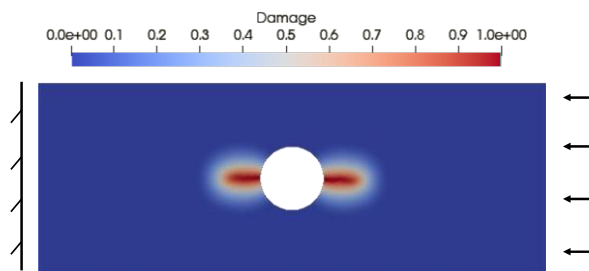
Figure 1: Damage field under compression using the new model [2].

**References:**

[1] E. Tanné, T. Li, B. Bourdin, J.-J. Marigo, C. Maurini. 2018. Crack nucleation in variational phase-field models of brittle fracture. J. Mech. Phys. Solids, 110: 80-99.

[2] L. De Lorenzis, C. Maurini. 2021. Nucleation under multi-axial loading in variational phase-field models of brittle fracture. Int. J. Frac.

[3] H. Amor, J.-J. Marigo, C. Maurini. 2009. Regularized formulation of the variation brittle fracture with unilater contact: numerical experiments. J. Mech. Phys. Solids, 57: 1209–1229.

[4] F. Freddi, G. Royer-Carfagni. 2010: Regularized variational theories of fracture: a unified approach. J. Mech. Phys. Solids, 58: 1154–1174.

# Stochastic phase-field modeling of brittle fracture

T. Gerasimov [1], U. Römer [2], J. Vondrejc [3], H. Matthies [3], L. De Lorenzis [4]

[1] Institut für Angewandte Mechanik, Technische Universität Braunschweig, Germany
[2] Institut für Dynamik und Schwingungen, Technische Universität Braunschweig, Germany
[3] Institut für Wissenschaftliches Rechnen, Technische Universität Braunschweig, Germany
[4] Department of Mechanical and Process Engineering, ETH Zürich, Switzerland

ldelorenzis@ethz.ch

**Abstract**: The phase-field modeling approach to fracture has recently attracted a lot of attention due to its remarkable capability to naturally handle fracture phenomena with arbitrarily complex crack topologies in three dimensions. On one side, the approach can be obtained through the regularization of the variational approach to fracture, which is conceptually related to Griffith's view of fracture; on the other side, it can be constructed as a gradient damage model with some specific properties.

The functional to be minimized is not convex, so that the necessary stationarity conditions of the functional may admit multiple solutions. The solution obtained in an actual computation is typically one out of several local minimizers. Evidence of multiple solutions induced by small perturbations of numerical or physical parameters was occasionally recorded but not explicitly investigated in the literature.

In this talk, based on the investigation in [1], the focus is placed on the issue of multiple solutions. Here a paradigm shift is advocated, away from the search for one particular solution towards the simultaneous description of all possible solutions (local minimizers), along with the probabilities of their occurrence. We propose the stochastic relaxation of the variational brittle fracture problem through random perturbations of the functional and introduce the concept of stochastic solution represented by random fields. The final result of the computation is not a single crack pattern, but rather several possible crack patterns and their probabilities.

**References:**

[1] T. Gerasimov, U. Römer, J. Vondrejc, H.G. Matthies, L. De Lorenzis (2020), Stochastic phase-field modeling of brittle fracture: computing multiple crack patterns and their probabilities. Computer Methods in Applied Mechanics and Engineering, 372: 113353.

# CC-FFM applied to LEBIM by two different approaches: curves method and PMTE-SC

M. Muñoz-Reja, V. Mantič, L. Távara

Grupo de Elasticidad y Resistencia de Materiales, Escuela Técnica Superior de Ingeniería, Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain
mmunozreja@us.es

**Keywords:** *LEBIM; CC-FFM; PMTE-SC; double cantilever beam test; composite joints*

**Abstract**: The Coupled Criterion of Finite Fracture Mechanics (CC-FFM) has been used by the present authors, in previous works, to predict the crack onset and/or growth along a linear-elastic interface modelled by means of the Linear Elastic Brittle Interface Model (LEBIM) [1].

The main objective of the present research is to analyse the relationship between two approaches associated to the CC-FFM: (a) the widely used method based on looking for the intersection of stress and energy criteria curves (referred hereinafter as the curves method) introduced by Leguillon [2] and Cornetti et al. [3]; and (b) the novel Principle of Minimum Total Energy subjected to a Stress Condition (PMTE-SC) introduced by Mantič [4]. For this purpose, four analytical models of the Double Cantilever Beam (DCB) test based on the Euler-Bernoulli beam model are developed. The DCB test is studied under two different boundary conditions (load and displacement control), by the curves method and the PMTE-SC. The obtained analytical results show that the behaviour of DCB test under load control and displacement control is very different and for this reason, the prediction of the CCFFM + LEBIM is different too. Furthermore, these analytical studies prove that the predictions by the curves method and the PMTE-SC are essentially identical.

**References:**

[1] M. Muñoz-Reja, L. Távara, V. Mantič, P. Cornetti. 2016. Crack onset and propagation at fibre-matrix elastic interfaces under biaxial loading using finite fracture mechanics. Composites Part A, 82, 267–278.

[2] D. Leguillon. 2002. Strength or toughness? a criterion for crack onset at a notch. European Journal of Mechanics A/Solids, 21, 61–72

[3] P. Cornetti, V. Mantič, A. Carpinteri. 2012. Finite Fracture Mechanics at elastic interfaces. International Journal of Solids and Structures 42, 1022-1032.

[4] V. Mantič. Prediction of initiation and growth of cracks in composites. 2014. Coupled stress and energy criterion of the finite fracture mechanics. 16th European Conference on Composite Materials (ECCM 2014)

# Using the coupled criterion to better understand architectural bio-inspired materials

Thomas Duminy [1], Aurélien Doitrand [1], Sylvain Meille [1]

[1] Univ Lyon, INSA Lyon, UCBL, CNRS, MATEIS, UMR5510, 69621 Villeurbanne, France
thomas.duminy@insa-lyon.fr

**Keywords:** *Coupled criterion, crack initiation, nacre-like alumina*.

**Abstract**: Nacre-like alumina (NLA) is a full-ceramic composite developed during the last decade that can potentially be used for load-bearing applications in harsh environment. It is composed of 95.0 vol% alumina platelets aligned preferentially in one direction and embedded in 5.0 vol% of a silica-calcite-alumina glassy phase to form a so-called 'brick-and-mortar' anisotropic structure that can exhibit extensive crack deflection when the cracks propagate along platelet thickness, resulting in stable crack growth (see Figure 1 Left).
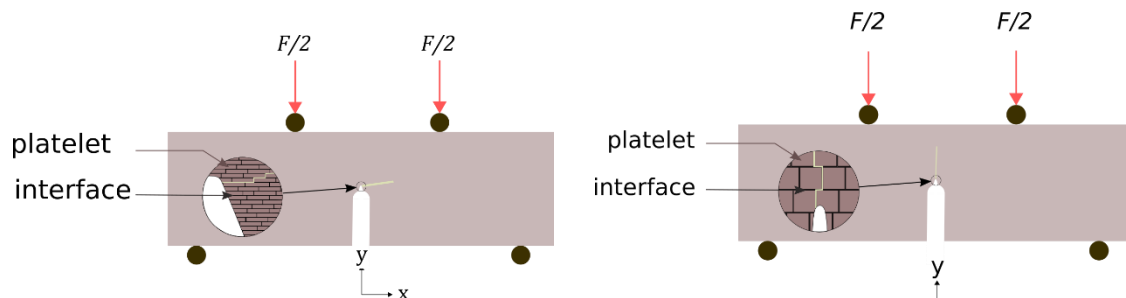
*Figure 1 - Nacre-like alumina tested under 4-points bending in two different orientations: (Left) Configuration with extensive crack deflection, and (Right) Configuration studied with an elastic brittle behavior*

Such a configuration shows an important crack propagation resistance and has been widely studied, both experimentally [1] and numerically [2].

However, to pave the way for the industrialization of NLA, one must also consider the mechanical behavior under other testing material orientations, not necessarily showing the most extensive crack deflection. In our study, we have tested the propagation at 90° from the previous configuration (see Figure 1 Right), with a crack propagation along the platelet width. Testing in such orientation leads to an elastic behavior with a brittle fracture. Limited crack deviation is still noted, with the presence of both broken platelets and platelets by-passed by the crack. SEM and X-ray tomographic characterization of the crack path were performed.

In terms of simulation, the use of finite fracture mechanics models such as the coupled criterion (CC) benefits from several advantages. First, experimental standards for toughness determination do not consider crack growth through a tortuous crack path. This may lead to poor estimation of material fracture parameters for nacre like alumina. Crack initiation at the interface between platelets is assessed using 2D or 3D application of the CC and compared to experimental results from micro-beam or macroscopic bending specimens.

Relationships between the macroscopic properties of NLA, the individual properties of its components and the microstructure could be determined. A better understanding of these relationships as well as of the crack initiation process can ultimately help the microstructure optimization to withstand complex loading and the triggering of stable crack growth.

**References:**

[1] H. Saad *et al*. 2020. A simple approach to bulk bioinspired tough ceramics. Materialia, volume 12:100807.

[2] A. Doitrand *et al*. 2020. Determination of interface fracture properties by micro and macro-scale experiments in nacre-like alumina. JMPS. Volume 145::104143.

# Multi-physics phase field fracture: from battery degradation to hydrogen embrittlement

Emilio Martínez-Pañeda[1]

[1] Department of Civil and Environmental Engineering, Imperial College London, London SW7 2AZ, UK

e.martinez-paneda@imperial.ac.uk

**Keywords:** *phase field; hydrogen embrittlement; fracture; fatigue; Li-Ion batteries*

**Abstract**: Phase field fracture methods have gained remarkable popularity in recent years. The reasons are arguably twofold. Firstly, phase field modelling has provided a suitable platform for the simple yet rigorous fracture thermodynamics principles first presented by Griffith. Crack propagation is predicted based on a global energy minimization, without the need for ad hoc criteria. Secondly, phase field formulations have proven to be computationally compelling; advanced fracture features such as complex crack trajectories, crack branching, nucleation, and merging can be captured robustly in arbitrary geometries and dimensions. Moreover, phase field approaches are well-suited to open new modelling horizons in multi-physics problems; the phase field evolution law can be readily combined with equations describing various coupled physical phenomena.

In this talk, I will review the fundamentals of phase field fracture methods and present some of our recent work in developing phase field formulations for coupled chemo-mechanical problems of notable technological relevance. Emphasis will be placed on phase field models for hydrogen assisted fracture [1-2] and fatigue [3], localized corrosion damage [4], and Li-Ion battery electrode particle cracking [5]. The predictive capabilities of these formulations will be showcased by benchmarking against experimental data. Large-scale case studies of engineering interest will also be addressed to demonstrate the potential of phase field-based modelling in enabling *Virtual Testing* in the energy sector.

**References:**

[1] E. Martínez-Pañeda, A. Golahmar, C.F. Niordson. *A phase field formulation for hydrogen assisted cracking*. Computer Methods in Applied Mechanics and Engineering 342, pp. 742-761 (2018)

[2] M. Isfandbod, E. Martínez-Pañeda. *A mechanism-based multi-trap phase field model for hydrogen assisted fracture*. International Journal of Plasticity 144, 103044 (2021)

[3] A. Golahmar, P.K. Kristensen, C.F. Niordson, E. Martínez-Pañeda. *A phase field model for hydrogen-assisted fatigue*. International Journal of Fatigue (in press)

[4] C. Cui, R. Ma, E. Martínez-Pañeda. *A phase field formulation for dissolution-driven stress corrosion cracking*. Journal of the Mechanics and Physics of Solids 147, 104254 (2021)

[5] W. Ai, B. Wu, E. Martínez-Pañeda. *A multi-physics phase field formulation for modelling fatigue cracking in lithium-ion battery electrode particles* (submitted)

# 3D Finite Fracture Mechanics under mode I loading: the flat elliptical crack

Pietro Cornetti [1], Vladislav Mantič [2]

[1] Department of Structural, Building and Geotechnical Engineering, Politecnico di Torino, Italy
[2] Group of Elasticity and Strength of Materials, School of Engineering, University of Seville, Seville, Spain
pietro.cornetti@polito.it

**Keywords:** *LEFM, FFM, elliptical cracks*.

**Abstract**: Finite Fracture Mechanics (FFM) relies on the assumption that cracks, at least at onset, grow in a finite, discrete way, i.e. by crack steps. FFM has proven to be an effective fracture criterion for predicting crack onset stresses in 2D geometries. Some attempts have been done to extend the criterion to 3D geometries [1]. The main difficulty lies in the fact that the failure stress depends not only on the size of the crack step, but also on its shape. Under mode I loading conditions, the dependence on the shape has no effect in 2D geometries while it has in 3D.

In order to have a first insight to the problem, we focus the analysis to flat elliptical cracks under uniform remote stresses normal to the crack plane. The problem can be seen as a straightforward generalization of the penny shaped crack geometry recently faced by FFM in [2]. We first summarize the classical results from the theory of elasticity for such a geometry. Then we apply LEFM showing that, even for an infinitesimal crack extension, the fracture stress depends on the shape of the crack growth; the shape providing the minimum failure stress generally is not the iso-stress one. Finally, we apply FFM. As expected and differently from LEFM, FFM is able to catch the size effect. For sufficiently small cracks (or less brittle materials), the crack step is determined by the iso-stress condition, while for large cracks (or highly brittle materials) the crack step is the one maximizing the energy release, as for the LEFM approach.

## References:

[1] A. Doitrand, D. Leguillon (2018) 3D application of the coupled criterion to crack initiation prediction in epoxy/aluminum specimens under four point bending. International Journal of Solids and Structures, 143: 175-182.

[2] P. Cornetti, A. Sapora (2019) Penny-shaped cracks by Finite Fracture Mechanics. International Journal of Fracture 219: 153–159.

# Variational simulation of failure in composites:
# an overview of the achievements based on the combination of the phase field approach to fracture and the cohesive zone model

Marco Paggi [1], José Reinoso [2], I.G. García [2], A. Quintana-Corominas [3], A. Turón [3],
F. Fantoni [4], A. Bacigalupo [5]

[1] IMT School for Advanced Studies Lucca, P.zza San Francesco 19, 55100 Lucca, Italy
[2] Group of Elasticity and Strength of Materials, School of Engineering, University of Seville, Camino de los Descubrimientos s/n, 41092, Seville, Spain
[3] AMADE, Polytechnic School, Universitat de Girona, Campus Montilivi s/n, 17071 Girona, Spain
[4] DICATAM, Università degli Studi di Brescia, via Branze 43, 25123 Brescia, Italy
[5] DICCA, Università degli Studi di Genova, via Montallegro 1, 16145 Genova, Italy

marco.paggi@imtlucca.it

**Keywords:** *composites, failure analysis, phase field fracture, cohesive zone model, finite element method*.

**Abstract**: This lecture will provide an overview of the recent achievements in the area of composite materials and structures based on the variational approach combining the phase field method for brittle fracture in the bulk and the cohesive zone model for the simulation of decohesion of existing interfaces. Results stem from the original idea proposed in [1] and further developed through extensive international co-operations with researchers from University of Seville, University of Girona, University of Porto, University of Genoa and University of Brescia. Examples regard micro-mechanical simulations of failure mechanisms in laminates [2] and in long fiber reinforced composites with a single or two fibers [3]; high performance computing simulations for realistic multiple fiber problems [4]; structural delamination induced by matrix cracking [5]; and innovative asymptotic homogenization schemes for the upscaling of finite element simulations in the case of diffuse damage [6]. Further directions of research are finally outlined.

**References:**

[1] M. Paggi, J. Reinoso. 2017. Revisiting the problem of a crack impinging on an interface: a modeling framework for the interaction between the phase field approach for brittle fracture and the interface cohesive zone model. Computer Methods in Applied Mechanics and Engineering, 321:145-172.

[2] V. Carollo, J. Reinoso, M. Paggi. 2018. Modeling complex crack paths in ceramic laminates: A novel variational framework combining the phase field method of fracture and the cohesive zone model. Journal of the European Ceramic Society, 38:2994-3003.

[3] T. Guillen-Hernandez, I.G. Garcia, J. Reinoso, M. Paggi. 2019. A micromechanical analysis of inter-fiber failure in long reinforced composites based on the phase field approach of fracture combined with the cohesive zone model. International Journal of Fracture, 220:181-203.

[4] T. Guillén-Hernández, A. Quintana-Corominas, I.G. García, J. Reinoso, M. Paggi, A. Turón. 2020. In-situ strength effects in long fibre reinforced composites: A micro-mechanical analysis using the phase field approach of fracture, Theoretical and Applied Fracture Mechanics 108, 102621.

[5] A. Quintanas-Corominas, A. Turon, J. Reinoso, E. Casoni, M. Paggi, J.A. Mayugo. 2020. A phase field approach enhanced with a cohesive zone model for modeling delamination induced by matrix cracking, Computer Methods in Applied Mechanics and Engineering 358, 112618.

[6] F. Fantoni, A. Bacigalupo, M. Paggi, J. Reinoso. 2020. A phase field approach for damage propagation in periodic microstructured materials, International Journal of Fracture, 223:53-76.

# On the effect of residual strength in debonding of direct shear tests

A. M. Mirzaei, P. Cornetti, A. Sapora, M. Corrado

Department of Structural, Geotechnical and Building, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

amir.mirzaei@polito.it

**Keywords:** *Analytical modeling; Debonding; Composite joints; Cohesive zone model; Finite fracture mechanics.*

**Abstract**: In the present study, the shear lag model is combined with three different bond-slip cohesive laws as well as a fracture mechanics-based approach to investigate the mechanics of debonding in direct shear tests by considering the effect of residual strength. Results illustrate the considerable impact of friction, even at debonding onset. It is demonstrated that debonding load of experiments with different bond lengths and widths can be predicted using the interface mechanical properties extracted from one geometry. Results show that despite the simplicity of equations and differences in bond-slip laws, all the models predict the debonding load with acceptable accuracy.
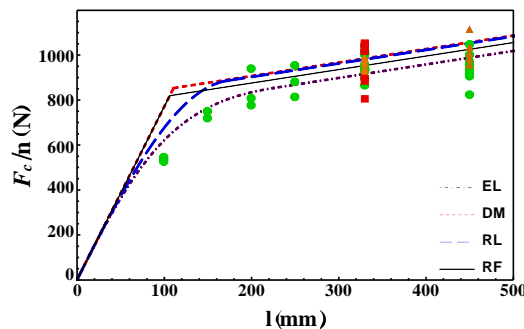
**Problem definition**:

It is seen that during experimental testing of direct shear tests, e.g., Fiber-Reinforced Cementitious Matrix (FRCM) to concrete joint or pull-out test, the load-displacement curve does not fall to zero but to a constant value, due to the friction that is modeled here as residual strength. In this investigation, four different models called "Equivalent- Linear Elastic Brittle Interface Model (EL), Dugdale Model (DM), Rigid-Linear Softening Model (RL), and Rigid-Finite Fracture Mechanics Model (RF)" are used to determine debonding mechanism. Furthermore, predictions of maximum debonding load are presented using closed-form equations as follows:

$$\bar{F}_c = \frac{F_c}{F_c^\infty} = \begin{cases} \frac{1}{1-\tau_r} \tanh\left[\lambda(1-\tau_r)\right], & \lambda \le \frac{1}{(1-\tau_r)} \operatorname{arccosh}\left[\frac{\sqrt{\tau_r}}{\sqrt{\tau_r}}\right] \\ \frac{1}{\sqrt{1-\tau_r}} + \tau_r(\lambda-\lambda_{lim}), & \lambda > \frac{1}{(1-\tau_r)} \operatorname{arccosh}\left[\frac{1}{\sqrt{\tau_r}}\right] \end{cases} \quad \text{EL}$$

$$\bar{F}_c = \frac{F_c}{F_c^\infty} = \begin{cases} \lambda, & \lambda \le \frac{1}{\sqrt{1-\tau_r}} \\ \frac{1}{\sqrt{1-\tau_r}} + \tau_r(\lambda-\lambda_{eff}), & \lambda > \frac{1}{\sqrt{1-\tau_r}} \end{cases} \quad \text{DM}$$

$$\bar{F}_c = \frac{F_c}{F_c^\infty} = \begin{cases} \frac{1}{1-\tau_r} \sin\left[\lambda(1-\tau_r)\right], & \lambda \le \frac{\arccos[\tau_r]}{1-\tau_r} \\ \sqrt{\frac{1+\tau_r}{1-\tau_r}} + \tau_r(\lambda-\lambda_{eff}), & \lambda > \frac{\arccos[\tau_r]}{1-\tau_r} \end{cases} \quad \text{RL}$$

$$\bar{F}_c = \frac{F_c}{F_c^\infty} = \begin{cases} \lambda, & \lambda \le 1 \\ 1+\tau_r(\lambda-1), & \lambda > 1 \end{cases} \quad \text{RF}$$

**Results**:

To compare the accuracy of models, maximum debonding loads reported in the literature [1] for the pull-push test of FRCM-to-concrete joints are used.



Maximum debonding load for different bond lengths and widths.

**References:**

[1] D'Antino T, Carloni C, Sneed LH, Pellegrino C. Matrix-fiber bond behavior in PBO FRCM composites: A fracture mechanics approach. Eng Fract Mech 2014;117:94–111.

# Three-point bending test and Digital Image Correlation (DIC) for determining cortical bone fracture toughness
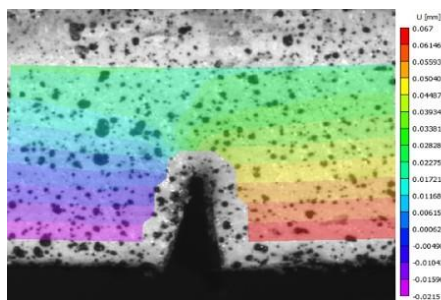
Maxime Levy [1]

[1] School of Mechanical Engineering, The Iby and Aladar Fleischman Faculty of Engineering, Tel Aviv University, Tel Aviv, 69978 Israel

maximelevy@mail.tau.ac.il

**Keywords:** *Long bone fracture*, phase field method, fracture toughness, Digital Image Correlation.

**Abstract**: Bone fracture prediction by CT-based Finite Element Analysis has been studied in [1] based on maximum principal strain criterion. This simplified criterion can predict fracture initiation load within 20% error (conservative prediction). It cannot predict the crack path which may also be of interest for clinical application. In an attempt to improve bone fracture prediction, we apply experimental methods to determine the material properties of bone tissue so to be used in conjunction with FEA and Phase Field Models (PFMs). So far, we focus on human femoral cortex and its failure in transverse direction (perpendicular to osteons). Cortical bone is a heterogeneous and transversely isotropic material. One of the key material parameters of PFM is the fracture toughness or more precisely the critical Energy Release Rate (ERR) $G_c$. In [2] PFM was studied as a feasible criterion to predict humerus fracture based on CT images with an assumed power-law correlation between the heterogeneous $G_c$ and Young modulus.

The objective is to provide experimental results for cortical bone fracture toughness in the transverse direction to evaluate its distribution in the bone cortex and to investigate a correlation with other material properties such as Young modulus and bone density. Since no standards are available for the determination of bone fracture toughness, we used the standards for metal and concrete to measure the critical Stress Intensity Factor (SIF) $K_c$ by a three-point bending test setup. These standards have been used in previous works ([3,4]) but we apply Digital Image Correlation (DIC) techniques to estimate the Crack Opening Displacement involved in $K_c$ determination. First bone specimens have been prepared and tested; the results obtained with the different standards are ranged from 2 to 9 $MPa\sqrt{m}$. Although it is consistent with the previous work ([3,4]), we need to improve the specimen preparation and the DIC procedure. Also, we need to test specimens from a bone which has been scanned by a clinical CT to compare $K_c$ values with density, Young modulus, and yield/ultimate strains.

Horizontal displacement from DIC



Three-point bending setup with DIC

**References:**

[1] Z. Yosibash, R. Plitman Mayo, G. Dahan, N. Trabelsi, G. Amir, C. Milgrom. 2014. Predicting the stiffness and strength of human femurs with real metastatic tumors. Bone, volume 69: pages 180-190.

[2] R. Shen, H. Waisman, Z. Yosibash, G. Dahan. 2019. International Journal for Numerical Methods in Biomedical Engineering, volume 35.

[3] P. Zioupos, J.D Currey. 1998. Changes in the Stiffness, Strength, and Toughness of Human Cortical Bone With Age. Bone, volume 22: pages 57-66

[4] A. Carpinteri, F. Berto, G. Fortese, C. Ronchei, D. Scorza, S. Vantadori. 2017. Modified two-parameter fracture model for bone. Engineering Fracture Mechanics, volume 174: pages 44-53

# Crack nucleation and variational phase-field models of fracture

Blaise Bourdin [1,2]

[1] Department of Mathematics & Statistics, McMaster University, Hamilton, ON, Canada
[2] Department of Mechanical & Industrial Engineering, Louisiana State University, Baton Rouge, LA, USA

bourdin@mcmaster.ca

**Keywords:** *Phase-fields fracture, nucleation*.

**Abstract**: This talk deals with crack nucleation in nominally brittle materials. I will start by recalling fundamental incompatibility between Griffith's theory and nucleation criteria based on a stress yield surface, the *strength vs. toughness* paradox.
Then, I will focus on three recent approaches [1-3] leveraging variational phase-field models of fracture. I will discuss the respective strengths, weaknesses, and open issues of each approach.

**References:**

[1] Kumar, A., Bourdin, B., Francfort, G. A., and Lopez-Pamies, O. (2020). Revisiting nucleation in the phase-field approach to brittle fracture. *J. Mech. Phys. Solids*, 142:104027.

[2] Tanné, E., Li, T., Bourdin, B., Marigo, J.-J., and Maurini, C. (2018). Crack nucleation in variational phase-field models of brittle fracture. *J. Mech. Phys. Solids*, 110:80–99.

[3] De Lorenzis, L. and Maurini, C. (2021). Nucleation under multi-axial loading in variational phase-field models of brittle fracture. *Int. J. Fracture*.

# A humidity dose-CZM formulation to simulate new end-of-life recycling methods for photovoltaic laminates

Zeng Liu [1,2], Jose Reinoso [2], Marco Paggi [1]

[1] IMT School for Advanced Studies Lucca, Piazza San Francesco 19, 55100, Lucca, Italy
[2] Elasticity and Strength of Materials Group, School of Engineering, University of Seville, Camino de los Descubrimientos s/n, 41092, Seville, Spain

zeng.liu@imtlucca.it

**Keywords:** *Cohesive zone model, Adhesion strength, Humidity dose model, Photovoltaic recycling*.

**Abstract**: It is well known that humidity and temperature greatly influence the degradation of interfacial adhesion in photovoltaic (PV) modules [1][2]. Besides, for accurate prediction of the required energy to peel off the different plies of end-of-life PV for recycling, it is also essential to take these factors into account. In this work, a polynomial cohesive zone model (CZM) coupled with a humidity dose model is proposed to address this in the finite element (FE) framework. A novel three-dimensional interface finite element considering large deformation is adopted to accurately deal with the coupled material and geometrical nonlinearity involved in peeling tests. A consistent derivation and operator formulations for this interface element are detailed in this work. Consistency between numerical predictions and peeling experimental results taken from literature [3] confirms the validity of the proposed approach. From the technical viewpoint, the numerical results show that peeling can be energetically preferable over crushing to disassamble and recycle PV laminates at the end of their lifetime. The proposed modeling approach can effectively contribute to virtually design new methods for PV recycling.

**References:**

[1] Polverini, D., Field, M., Dunlop, E. and Zaaiman, W., 2013. Polycrystalline silicon PV modules performance and degradation over 20 years. Progress in photovoltaics: research and applications, 21(5), pp.1004-1015.

[2] McIntosh, K.R., Powell, N.E., Norris, A.W., Cotsell, J.N. and Ketola, B.M., 2011. The effect of damp-heat and UV aging tests on the optical properties of silicone and EVA encapsulants. Progress in Photovoltaics: Research and Applications, 19(3), pp.294-300.

[3] Wu, D., Zhu, J., Betts, T.R. and Gottschalg, R., 2014. Degradation of interfacial adhesion strength within photovoltaic mini-modules during damp-heat exposure. Progress in Photovoltaics: Research and Applications, 22(7), pp.796-809.

# Thermoelastic fracture analysis of thin-walled structures relying on the solid shell concept

P.K.Asur Vijaya kumar [1,2], A.Dean[2,3], J.Reinoso [2], M.Paggi[1]

[1] IMT School for Advanced Studies, Lucca, Piazza San Francesco 19, 55100, Lucca
[2] School of Engineering, Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092, Seville, Spain Full
[3] School of Civil Engineering, Sudan University of Science and Technology, PO Box 72 Eastern Daim, Khartoum, Sudan.

pavan.asur@imtlucca.it

**Keywords:** *Phase-Field, Solid Shell,* Thermo-Mechanical, Finite Element Methods

**Abstract**: Phase-field (PF) methods applied to fracture have become attractive modeling tools to solve complex problems of triggering cracking events in solids. These numerical techniques have significantly mitigated well-known mesh-dependent pathologies of standard FE-based methods for fracture, and are widely applied to brittle, ductile, and even plastic deformation models.

 Fracture of thin-walled structured components has been a matter of intensive research in the last decades. The occurrence of these phenomena notably restricts the study of the engineering systems, especially under the influence of temperature. With the aim of achieving reliable prediction tools for temperature coupled fracture in thin-walled structures, this research presents a thermodynamically consistent framework for the coupled thermo-mechanical phase-field model for thin-walled structures using a fully-integrated solid shell. The kinematic description of the body is constructed using the solid shell concept. This enables the use of three-dimensional constitutive thermo-mechanical models for the definition of material. The proposed thermo-mechanical phase-field models integrate with enhanced assumed strain (EAS) to alleviate Poison and volumetric locking pathologies. It is further combined with the Assumed natural strain (ANS) method leading to a locking-free thermo-mechanical solid shell phase-field element using a fully integrated interpolation scheme.

From the formulation standpoint, the fully coupled thermo-elastic Kirchhoff-Saint-Venant material model is used to describe the Helmholtz free energy of the thermo-mechanical system in bulk, further combined with the free energy (fracture energy) stemming from the phase field. Therefore, the total internal energy functional is modeled as an amalgamation of:
  (i)     total elastic energy constituting from bulk (along with additive decomposition of enhanced stains incorporated into the incompatible strain field due to EAS).
  (ii)    thermal energy stemming from the thermo-elastic material model and its constitutive couplings.
  (iii)   surface energy (Crack energy) stemming from the fracture of the shell.

The emphasis is further made on evaluating the corresponding thermodynamic consistency and the variational formalism leading to the non-linear coupled equations equipped with the coupled driving force. These coupled equations are numerically solved using finite element methods emphasizing the implementation by utilizing the user-defined capability UEL of *ABAQUS*.

Several representative examples are examined to assess the practicability and reliability of the proposed modeling framework.

**References:**

[1] J. Reinoso, M.Paggi, C.Linder (2017), "*Phase field modelling of brittle fracture for enhanced assumed tsrain shells at large deformations: formulation and finite element implementation*", Comput Mech, 59:981-1001.

[2] P K Asur Vijaya Kumar, A.Dean, S.Sahraee, J.Reinoso and M.paggi (2021): *Non-linear thermoelastic and fracture analysis of thin-walled structures with cohesive-like interface relying on the solid shell concept*, Under review in FINEL.

# Semi-analytic computation of singular elastic solutions in anisotropic multimaterial corners

M.A. Herrera-Garrido[1], V. Mantic[1], A. Barroso[1]

[1] School of Engineering, University of Sevilla, Group of Elasticity and Strength of Materials, Camino de los Descubrimientos s/n, Seville E-41092, Spain
mherrera13@us.es

**Keywords:** *stress singularities, multimaterial corners, anisotropic elasticity, friction, Stroh formalism*

**Abstract**: A computational code developed in MATLAB to compute singularity exponents and the singular stresses and displacement fields in multimaterial corners under general plane strain has been developed. In this work we present the code whose input data are the elasticity coefficients and geometry of each material wedge that conform the corner and the boundary and interface conditions. The considered homogeneous boundary conditions are stress free, clamped faces, faces with some restricted or allowed displacement direction, either in their plane or in an inclined plane, or with frictional sliding. The interface conditions between two consecutive materials covered by the software are perfectly bonded and frictionless or frictional sliding interfaces. This code can analyze open and closed (periodic) corners, including isotropic materials, and transversely isotropic and orthotropic materials with an arbitrary spatial orientation.

This code is based on the Stroh sextic formalism for anisotropic elasticity [1,2,3,4], the concept of transfer matrix for single material wedges proposed by T.C.T. Ting [5] and a suitable matrix formalism for the considered boundary and interface conditions [6,7].

This code has been successfully verified by comparing the present results with the results obtained by analytic eigen-equations or computational codes by many authors. The code is a general computational tool that can help other researchers in several ways. First, knowing the singularity exponents at singular points (crack/corner vertex) could help researchers using FEM allowing them to get more accurate results near corners and specially to predict fracture initiation there. Second, many researchers deduce their own analytic eigen-equations for specific cases with no easy way to check if their results are correct, but by comparing their results with those obtained by the present general computational tool could be a fast and reliable way to corroborate their equations.

**References:**

[1] A.N. Stroh. 1958. Dislocations and cracks in anisotropic elasticity. Philosophical Magazine, 3 (30): 625–646.
[2] A.N. Stroh. 1997. Steady state problems in anisotropic elasticity. Journal of Mathematics and Physics. 41: 77–103.
[3] A. Barroso, V. Mantič, F. París. 2003. Singularity analysis of anisotropic multimaterial corners. Int. J. Fracture. 119: 1-23
[4] T.C.T. Ting. 1996. Anisotropic Elasticity: Theory and Applications. Oxford University Press
[5] T.C.T. Ting. 1997. Stress singularities at the tip of interfaces in polycrystals. Damage and Failure of Interfaces, edited by Rossmanith, 75–82. Rotterdam: Balkema Publishers.
[6] V. Mantič, F. París, J. Cañas. 1997. Stress singularities in 2D orthotropic corners. International Journal of Fracture, 83: 67–90.
[7] V. Mantič, A. Barroso, F. París. 2014. Singular elastic solutions in anisotropic multimaterial corners. Applications to composites. Mathematical Methods and Models in Composites, edited by Mantič, 426-489.

# Simulation of bridging mechanisms in composite laminates using hybrid PF-CZM method

A.R. Dusane[1], P.R. Budarapu[2], M. Paggi[1], J. Reinoso[3]

[1] IMT School for Advanced Studies Lucca, 19 Piazza San Francesco, 55100 Lucca, Italy
[2] School of Mechanical Sciences, Indian Institute of Technology, Bhubaneswar 752050, India
[3] Elasticity and Strength of Materials Group, School of Engineering, Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092, Seville, Spain

ajinkya.dusane@imtlucca.it

**Abstract**: Delamination and cracking of matrix/fiber is a common failure phenomenon reported in fiber-reinforced composites. The complex state of stress developed in laminated structures; they are prone to fractures. Therefore, designs with large damage tolerance are currently implemented in most the industrial sectors. This can be achieved by designing the composites for failure, which requires a comprehensive understanding of failure mechanisms. Cohesive Zone Models (CZM) is a popular technique to study debonding and decohesion in composite structures. Furthermore, due to the accurate simulation of complex crack paths, including crack branching, the Phase Field (PF) approach has gained notable relevance in fracture studies [1], including the interplay between laminae-laminate debonding and crack propagation in the matrix [2, 3]. The present study focuses on crack growth in composite laminates using a hybrid PF-CZM approach. Bridging mechanisms in intralayer and interlayer and crack simulation coupling the phase field approach and the cohesive zone model is herein used for identifying crack migration through material layers.

The crack paths and the related force-displacement curves of 2D multilayered material models of complex laminates are predicted and compared. The numerical study is carried out for bi-layer, tri-layered and multi-layer, and woven composites considering critical fracture energies equal to 0.025 N/mm, 2.5 N/mm, and no interface, respectively. The study shows that introducing an interface improves the fracture toughness of the composite. It is observed that penetration vs. deflection events in composite depends on the properties of the interface. When the initial crack is considered within the material with low fracture toughness, crack is observed to travel across the interface, realized by crack deflection along with the interface. The influence of laminate thickness in crack propagation is also studied. The fracture toughness is observed to increase with the increase in the area of laminate having higher fracture toughness. The obtained results give new insights into understanding failure mechanisms in the laminated composites.

**References:**

[1] Christian Miehe, Martina Hofacker, and Fabian Welschinger. 2010. A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits. Computer Methods in Applied Mechanics and Engineering. 199(45-48):2765–2778.

[2] M. Paggi and J. Reinoso. 2017. Revisiting the problem of a crack impinging on an interface: A modeling framework for the interaction between the phase field approach for brittle fracture and the interface cohesive zone model. Computer Methods in Applied Mechanics and Engineering. 321:145–172.

[3] V. Carollo, J. Reinoso, and M. Paggi. 2018. Modeling complex crack paths in ceramic laminates: A novel variational framework combining the phase field method of fracture and the cohesive zone model. Journal of the European Ceramic Society. 38:2994–3003.

# Virtual Element Method for Micro Damage and Cracking in Fibre-Reinforced Composites

Marco Lo Cascio[1], Vladislav Mantič [1], Luis Távara [1]

[1] Escuela Técnica Superior de Ingeniería, Universidad de Sevilla
Camino de los Descubrimienos s/n, 41092 Sevilla, Spain

mlocascio@us.es

**Keywords:** *Virtual Element Method, Damage, Fibre-Reinforced Composites.*

**Abstract**: Computational methods, such as FEM, BEM, FEM/BEM, PFEM, X-FEM, SBFEM, among others, have been employed in the literature to model crack propagation processes [1]. With finite element techniques, crack propagation is modelled explicitly by consecutively adapting the spatial discretization of the domain. The simplest methods to simulate crack propagation restrict the crack propagation along the element edges, introducing a computed crack path dependence on the mesh topology. More powerful but complex methods may combine element modification techniques, such as element splitting, with advanced re-meshing strategies, in which the overall mesh is progressively updated starting from a finer mesh in the proximity of the crack tip region. Such a continuous mesh refinement process may generally affect large regions of the analysis domain due to the need to preserve a conforming transition between the re-meshed propagation region and the surrounding areas. It is thus a computationally intensive operation.

The Virtual Element Method (VEM) [2] is a recent generalization of FEM that can handle polygonal elements of arbitrary shapes. This distinctive feature allows avoiding mesh dependency of the crack propagation direction. Any computed crack path can be represented by modifying the topology of the virtual element over which the crack propagation occurs, including the crack edges as new element edges without the need for further re-meshing. This straightforward approach can be employed either when the computed crack length increment is large enough to split an element entirely or when it splits this element only partially. In both cases, the resulting element topology is valid from the VEM's standpoint and does not need any further mesh modification [3]. Moreover, to improve numerical accuracy near the crack tip, a local mesh refinement can be introduced by subdividing one or more of the local elements in any number of elements of arbitrary shape without necessarily affect large portions of the analyzed domain. A schematic of the modelling benefits offered by the VEM in crack propagation modelling is shown in Fig. 1. In this contribution, the proposed VEM approach is exploited to simulate the growth of kinked cracks originating from the partial fibre-matrix debonding in a fibre-reinforced composite material subjected to transverse load [4].
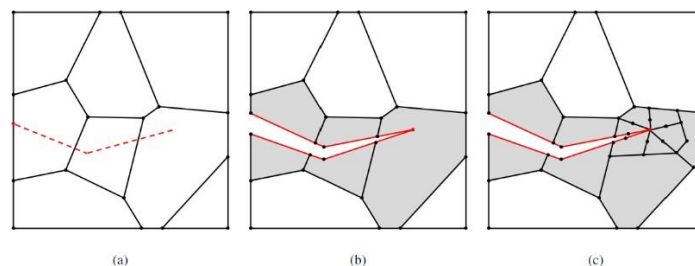
Fig. 1: Crack propagation with VEM: (a) computed crack path; (b) elements splitting; (c) local mesh refining.

**References:**

[1] M. Kuna. 2013. Finite elements in fracture mechanics. Springer, Dordrecht.

[2] L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, and A. Russo. 2013. Basic principles of virtual element methods. Mathematical Models and Methods in Applied Sciences, 23: 199–214.

[3] M. Lo Cascio, I. Benedetti, V. Mantič. 2020. Micro Damage and Cracking in Fibre Reinforced Composites by a Novel Hybrid Numerical Technique. AIP Conference Proceedings, 2309: 020001.

[4] F. París, E. Correa, and V. Mantič. Kinking of transversal interface cracks between fiber and matrix. 2007. Journal of Applied Mechanics, 74: 703-716.

# Phase-field modeling of brittle fracture in heterogeneous materials: a one-dimensional study

Francesco Vicentini [1], Pietro Carrara [1], Laura De Lorenzis [1]

[1] Computational Mechanics Group, ETH Zürich, Tannenstrasse 3, 8092 Zürich, Switzerland

fvicentini@ethz.ch

**Keywords:** *damage, fracture, heterogeneity, fracture toughness, strength, phase-field.*

**Abstract**: Phase-field modeling of brittle fracture, first proposed by Bourdin et al. [1] as regularization of Francfort and Marigo's variational approach to fracture mechanics [2] and later re-interpreted as a special family of gradient damage models [3], provides a remarkably flexible variational framework to describe the nucleation and propagation of cracks with arbitrarily complex geometries and topologies in two and three dimensions.

The approach is based on the assumption that the brittle material exhibits homogeneous elastic and fracture properties (fracture toughness). On the other hand, many brittle materials are characterized by strongly heterogeneous properties, one important category being biological tissues such as bones. Phase-field modeling of fracture in these tissues is particularly attractive due to their typically complex crack patterns, yet it requires the extension of the approach to the case of heterogeneous mechanical properties.

Previous studies addressing fracture in heterogenous materials have adopted a pragmatic approach, by simply substituting the constant fracture toughness of the original model with a fracture toughness depending on the material point [4, 5, 6, 7]. However, to the best of our knowledge the implications of heterogeneous material properties on the key predictions of the phase-field model have not been thoroughly investigated yet.

In this contribution, we start such investigation with the one-dimensional case. We revisit the fundamental mathematical analysis in [3] by assuming that the material properties are heterogeneous with different possible profile shapes. Our main goal is to quantitatively assess how the heterogeneity in material properties influences the fracture toughness and the tensile strength of a one-dimensional bar.

**References:**

[1] G. A. Francfort, J. J. Marigo. 1998. Revisiting brittle fracture as an energy minimization problem. Journal of the Mechanics and Physics of Solids, 46(8), pp.1319-1342.

[2] B. Bourdin, G. A. Francfort, J. J. Marigo. 2000. Numerical experiments in revisited brittle fracture. Journal of the Mechanics and Physics of Solids, 48(4), pp.797-826.

[3] J. J. Marigo, C. Maurini, K. Pham. 2016. An overview of the modelling of fracture by gradient damage models. Meccanica, 51(12), pp.3107-3128.

[4] P. A. V. Kumar, A. Dean, J. Reinoso, P. Lenarda, M. Paggi. 2021. Phase field modeling of fracture in Functionally Graded Materials: Γ-convergence and mechanical insight on the effect of grading. Thin-Walled Structures, 159, p.107234.

[5] S. Natarajan, R. K. Annabattula, E. Martínez-Pañeda. 2019. Phase field modelling of crack propagation in functionally graded materials. Composites Part B: Engineering, 169, pp.239-248.

[6] M. Z. Hossain, C. J. Hsueh, B. Bourdin, K. Bhattacharya. 2014. Effective toughness of heterogeneous media. Journal of the Mechanics and Physics of Solids, 71, pp.15-32.

[7] R. Shen, H. Waisman, Z. Yosibash, G. Dahan. 2019. A novel phase field method for modeling the fracture of long bones. International journal for numerical methods in biomedical engineering, 35(8), p.e3211.

# Numerical study of unfolding failure using Phase Field fracture method

Sindhu B.S [1], E. Graciani [2], B. López-Romano [1]

[1] FIDAMC, Foundation for the Research, Development and Application of Composite Materials,
Avda. Rita Levi Montalcini 29, 28906 Getafe, Madrid, Spain
[2] Grupo de Elasticidad y Resistencia de Materiales, Escuela Técnica Superior de Ingeniería,
Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain
Sindhu.bushpalli@fidamc.es

**Abstract**: Highly curved composite laminates used at the junction between two perpendicular panels experience unfolding failure. Unfolding failure is a delamination which occurs in the curved laminates when they are loaded under bending moment trying to flatten the curvature which induces out-of-plane stresses, causing different plies to separate. The interlaminar tensile strength is determined empirically by means of four point bending test of L-shaped unidirectional samples. Notwithstanding, many studies have noticed that four-point bending test, applied to curved multidirectional composite laminates, show a thickness dependance of interlaminar tensile strength with the thickness of the specimens. This thickness dependance is explained by the concept of induced unfolding (see, for instance, Gonzalez-Cantero [1]) which states that unfolding failure may be caused by both interlaminar and intralaminar stresses. In the first stage, an intralaminar crack is formed which, under a certain level of interlaminar stresses, propagates as a delamination. The current study is focused on analyzing the initial stage of unfolding failure mechanism, associated with the intralaminar stresses, using both numerical and experimental study.

Numerical analysis involves the study of failure mechanisms causing unfolding failure through finite element models in Abaqus using the available subroutines. Numerical simulations include using Phase Field (PF) fracture method for determining intralaminar failure, Finite Fracture Mechanics (FFM) for debonding failure and to prove the instability of multiple delaminations after the initial intralaminar failure. Currently, Linear Elastic Boundary Interface Model (LEBIM) formulation in 3D is considered for the FFM approach [2]. For the PF approach, readily available subroutines (UMAT & HETVAL) with their different formulations [3,4] are being used for this research.

In the current work, PF fracture methodology is primarily studied by recreating simple 2D models in Abaqus and using the user material subroutine HETVALg available from the literature [2]. Later, several tests are being performed to understand the contribution of various parameters on the crack onset and its propagation. Following this, the 2D model was modified by adding weaker plies to contain the damage within the weaker ply and was monitored for the failure behavior. The same problem was recreated using a 3D model to analyze the propagation of damage through the width of specimen.

Furthermore, the potential of this phase field fracture method is being analyzed by performing simulations on various 2D models (including corner radius specimens) combining the phase field elements with regular plain strain elements.

**References:**

[1] J.M. González-Cantero, E. Graciani, B. López-Romano, and F. París. 2018. Competing mechanisms in the unfolding failure in composite laminates. Compos Sci Technol, 156:223–230.

[2] M.Muñoz-Reja, L.Távara, V.Mantič, P.Cornetti. 2020. A numerical implementation of the Coupled Criterion of Finite Fracture Mechanics for elastic interfaces. Theor Appl Fract Mech, 108:102607.

[3] Y. Navidtehrani, C. Betegón, E. Martínez-Pañeda. 2021. A Unified Abaqus Implementation of the Phase Field Fracture Method Using Only a User Material Subroutine. Materials, 14(8):1913.

[4] Y. Navidtehrani, C. Betegon, E. Martinez-Pañeda. 2021. A simple and robust Abaqus implementation of the phase field fracture method. Appl Eng Sci, 6:100050.

# 2D Micromechanical analysis of a representative volume element (RVE) made of a thermoset composite material using Phase-Field fracture and cohesive interface damage.

Juan Macías [1], Albertino Arteiro [1], José Reinoso [2], Pedro Camanho [3], Fermín Otero [4]

[1] DEMec, Universidade do Porto, Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
[2] Elasticity and Strength of Materials Group, School of Engineering, Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092, Seville, Spain
[3] INEGI, Universidade do Porto, Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
[4] Department of Nautical Science and Engineering, UPC, Pla de Palau 18, 08003 Barcelona, Spain

up202009641@fe.up.pt (J. Macías)

**Keywords:** *Finite element method, Phase-Field, Micromechanics, Matrix cracking.*

**Abstract**: Intralaminar fracture is one of the first failure mechanisms that take place in composite laminates. In quasi-isotropic or cross-ply configurations, damage usually starts in the form of matrix cracks in the plies that are located transversal to the main load direction. Such cracks can propagate to other laminae and promote further damage in the form of delamination. Thus, predicting the nature of matrix dominated failure is of primal importance for the optimal design of composite structures. In this work, the Phase-Field (PF) method in conjunction with interface cohesive damage have been use to investigate the mechanisms involved in failure initiation and propagation in a representative volume element (RVE) made of a thermoset composite material under transverse loading using the finite element method (FEM). The RVE fibre's distribution was obtained using Digimat and the numerical model set using Abaqus, such that matrix failure is captured using the new material model subroutine for PF damage of Martínez-Pañeda et al. [1], [2]. Fibre-matrix decohesion is modelled using cohesive surface interactions. Fibre failure was not considered because it is not expected in this load scenario. This work focuses mainly on the analysis of the material and mesh parameters for the setup of the RVE numerical model. It was found that the combination of specific material properties can bias the selection of the PF formulation that could be used for the specific scale. Thus, large values of critical energy release rate may require length scale parameters that are larger than the representative micro-scale dimension. Therefore, reducing the length scale without disrupting the material strength can only be done with specific PF formulations. Mesh convergence was also studied using elements with linear and quadratic formulations. It was found that fracture localization and convergence using quadratic elements is less demanding in terms of minimum element size and thus less expensive because the number of nodes is inferior. Finally, it was found that with the proper RVE configuration, the model can accurately reproduce the initial elastic stage and the expected failure mechanisms. Besides, the failure stress is close to the material strength. These results could be easily misrepresented if the fibres inside the volume are misplaced, such as partially cut by the boundaries creating a pre-cracked effect and promoting accelerated failure.

**References:**

[1]     Y. Navidtehrani, C. Betegón, and E. Martínez-Pañeda, "A simple and robust Abaqus implementation of the phase field fracture method," *Appl. Eng. Sci.*, vol. 6, p. 100050, Jun. 2021, doi: 10.1016/J.APPLES.2021.100050.

[2]     Y. Navidtehrani, C. Betegón, and E. Martínez-Pañeda, "A Unified Abaqus Implementation of the Phase Field Fracture Method Using Only a User Material Subroutine," 2021, doi: 10.3390/ma14081913.

# Extension of Finite Fracture Mechanics to dynamic loading scenarios

A. Chao Correas, M. Corrado, A. Sapora, P. Cornetti

Department of Structural, Building and Geotechnical Engineering, Politecnico di Torino,
Corso Duca degli Abruzzi 24, 10129, Torino, Italy

arturo.chaocorreas@polito.it

**Keywords:** *Finite Fracture Mechanics, Dynamic loading, Time Incubation Failure criterion, Experiments*.

**Abstract:** In recent years, many implementations of the Finite Fracture Mechanics approach have been successfully undertaken for several quasi-static scenarios containing both stress concentrations and intensifications. Because of the showcased good performance, this work aims at extending the Finite Fracture Mechanics failure criterion for comprising also certain dynamic loading cases, while maintaining the validity for the static ones.

To that end, the general requirements of a proper dynamic failure criterion are herein first reasoned and shortlisted. Then, following these constraints, a proposal for the Dynamic extension of Finite Fracture Mechanics (DFFM) is made, compared against another dynamic failure criterion from the literature (the Incubation Time Criterion for failure proposed in [1]) and proven to be more robust. Eventually, the comparison of the DFFM failure predictions for Notched Semi-Circular Bend Tests (See Figure 1a) with proper experiments from [2] is undertaken, showcasing excellent agreement as seen in Figure 1b.

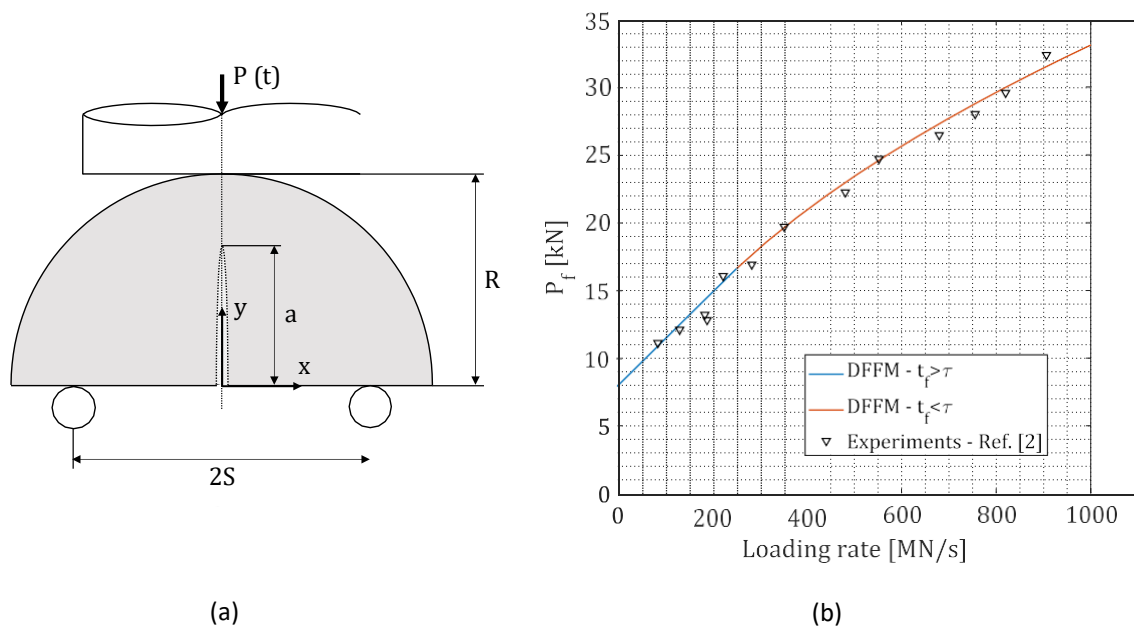|                                    |                                    |
| :--------------------------------: | :--------------------------------: |
| (a)                                | (b)                                |

Figure 1: (a) Schematic representation of a dynamic Notched Semi Circular Bend Test; (b) Comparison of the failure load predictions by DFFM and experimental results in [1] for several loading rates.

**References:**

[1] Y. V. Petrov, N.F. Morozov. 1994. On the Modelling of Fracture of Brittle Solids, J. Appl. Mech. 61, 710–712. https://doi.org/10.1115/1.2901518.

[2] W. Yao, K. Xia, T. Zhang. 2019. Dynamic Fracture Test of Laurentian Granite Subjected to Hydrostatic Pressure, Exp. Mech. 59, 245–250. https://doi.org/10.1007/s11340-018-00437-4.